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CONJUGATED HEAT TRANSFER OF FILM POOL BOILING ON A HORIZONTAL TUBE

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Abstract—The film pool boiling on a horizontal tube is analyzed as a conjugated heat transfer problem. A non-dimensional parameter, H, which was deduced from the governing differential equations, has been used to characterize the peripheral wall heat conduction of the heater. The results are implicitly compared with that of the Bromley solution.

INTRODUCTION

Given uniform heat generation within a heater placed in an asymmetrical fluid flow boundary condition, as in the case of a horizontally placed cylindrical heater in pool boiling, heat flows by conduction within the wall of the heater, creating a non-uniform wall surface temperature distribution. This is an example of conjugated heat transfer problems and such conjugated heat transfer problems have been increasingly investigated recently. Some of the studies on the problems for different aspects of heat transfer will be briefly mentioned.

Sunden (1980), who studied the heat transfer from a circular cylinder with a heated internal core region in low Reynolds number flow, showed that the ratio of the thermal conductivities of the solid body and fluid has a significant influence on the heat transfer rate.

Abramzon (1986), in his study of heat transfer between a solid spherical particle and a uniform laminar flow, also found that the ratio of the thermal conductivities of the material of the sphere and of the fluid was an important parameter which markedly affects both the local temperature and heat flux variation along the surface of the sphere.

Lee & Kakade (1976), who studied the effect of peripheral wall conduction on heat transfer from a cylinder in cross flow, concluded that the local heat transfer was strongly influenced by the local thermal conditions, which are affected by the physical dimensions and thermal properties of the heater.

Baughn (1978) has demonstrated analytically that a circumferential conduction number, which is a function of the wall properties and the boundary conditions, significantly affects the convection heat transfer in circular tubes.

Cess (1962), who used a very simple approximate analysis on the forced-convection film boiling on a flat plate with uniform surface heat flux, has shown that the Nusselt number was greater by a factor of 1.41 for a uniform surface heat flux as compared to a constant wall temperature.

Pool boiling heat transfer has been studied extensively for many years. The effects of fluid and thermal properties, of surface finish and coating, of orientation and geometry of the heater(s), of agitation of the working fluid, of the force field etc., have been investigated and a large number of correlations have been proposed. Many of the existing results on supposedly identical phenomena are inconsistent or differ widely from each other.

It is obvious that to compare the experimental results obtained by different investigators, all parameters governing the heat transfer process should be set equal. Seldom included is the effect of the variation of surface temperature, which is dependent on the Biot number, and the specific heat generation rate of the heater. A few studies on nucleate pool boiling heat transfer (Kovalev *et al.*, 1970; Berenson 1962; Magrini & Nannei 1975; Sauer *et al.* 1978; Jensen & Jackman 1984) indirectly recognize this variation of the surface temperature on the surface heat transfer coefficient.

A recent experimental study by Zeng & Lee (1987) showed that a non-dimensional circum-

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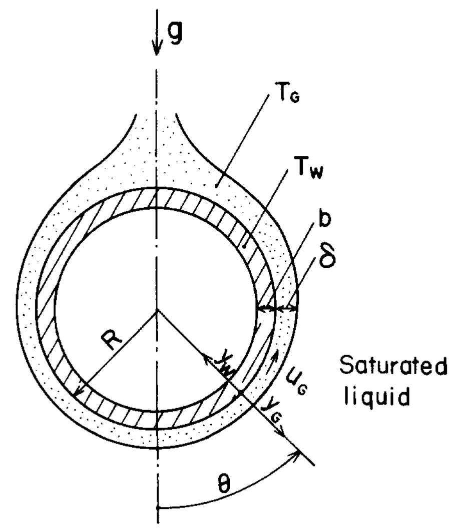

Figure 1. Idealized model.

ferential conduction parameter has a significant effect on the nucleate pool boiling heat transfer process.

Therefore, a study on the boiling heat transfer from a heater placed in an asymmetrical fluid flow condition must recognize the effect of the peripheral wall conduction on the overall heat transfer rate. However, no analytical study on film pool boiling heat transfer seems to exist which recognizes the effect of this peripheral wall conduction on heat transfer.

In the present study the film pool boiling on a horizontal tube is analyzed as a conjugated heat transfer problem. A non-dimensional parameter, H, which was deduced from the governing differential equations, has been used to characterize the peripheral wall heat conduction of the heater. The results are compared with that of Bromley's (1950) analytical solution with zero vapor velocity at the vapor–liquid interface for an isothermal horizontal cylinder.

ANALYSIS

The film pool boiling heat transfer from a long circular tube with a uniform specific heat generation, placed horizontally in a saturated liquid, is considered in the present study—as idealized in figure 1. The assumptions are as follows:

(i) In the vapor film region, the flow is steady and laminar and the heat transfer is by conduction only; i.e. the model of the vapor film used in the present analysis is essentially the same as that used by Bromley (1950).
(ii) Uniform heat generation within the tube wall and an insulated inside wall surface.
(iii) The effect of curvature of the tube is negligible.
(iv) All physical properties are constant.
(v) Radiation heat transfer is excluded.

Governing Equations and Boundary Conditions

(1) For the tube wall

$$\frac{1}{R^2}\frac{\partial T_w^2}{\partial \theta^2}+\frac{\partial^2 T_w}{\partial y_w^2}+\frac{\dot{q}}{K_w}=0. \qquad [1]$$

The boundary condition is

$$K_{\mathrm{w}} \frac{\partial T_{\mathrm{w}}}{\partial y_{\mathrm{w}}}\bigg|_{b} = 0, \qquad [2]$$

where T, $\dot{q}$ and K are the temperature, specific heat generation and thermal conductivity, respectively; R, b, θ and y are the outside radius of the tube, the wall thickness, the angle co-ordinate (figure 1) and the co-ordinate normal to the tube surface (figure 1); and the subscript w refers to the tube wall.

(2) For the vapor film

(a) Momentum equation:

$$\mu_{\mathrm{G}} \frac{\partial^2 u_{\mathrm{G}}}{\partial y_{\mathrm{G}}^2} + g(\rho_{\mathrm{L}} - \rho_{\mathrm{G}})\sin\theta = 0. \qquad [3]$$

(b) Energy equation:

$$K_{\mathrm{G}} \frac{\partial^2 T_{\mathrm{G}}}{\partial y_{\mathrm{G}}^2} = 0. \qquad [4]$$

The boundary conditions are

$$u_{\mathrm{G}}(0) = 0, \quad u_{\mathrm{G}}(\delta) = 0, \quad T_{\mathrm{G}}(\delta) = T_{\mathrm{sat}}, \qquad [5]$$

where u, δ and T_{sat} are the tangential velocity, vapor film thickness and saturation temperature, respectively; g, ρ and μ are the gravity acceleration, density and viscosity, respectively, and the subscripts G and L refer to the vapor film and the liquid, respectively.

(3) Interfacial boundary conditions

(a) Heat flux at the vapor–liquid interface:

$$-K_{\mathrm{G}} \frac{\partial T_{\mathrm{G}}}{\partial y_{\mathrm{G}}}\bigg|_{\delta} = \left[\frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}\theta}\int_0^{\delta} (\rho_{\mathrm{G}} u_{\mathrm{G}})\,\mathrm{d}y_{\mathrm{G}}\right] h_{\mathrm{LG}}, \qquad [6]$$

where h_{LG} is the latent heat of evaporation.

(b) Temperature and heat flux at the solid–vapor interface:

$$T_{\mathrm{w}}(0) = T_{\mathrm{G}}(0); \quad K_{\mathrm{w}} \frac{\partial T_{\mathrm{w}}}{\partial y_{\mathrm{w}}}\bigg|_{0} = -K_{\mathrm{G}} \frac{\partial T_{\mathrm{G}}}{\partial y_{\mathrm{G}}}\bigg|_{0}. \qquad [7]$$

(c) Energy balance at the solid–vapor interface:

$$\int_0^{\pi} \left(K_{\mathrm{w}} \frac{\partial T_{\mathrm{w}}}{\partial y_{\mathrm{w}}}\bigg|_{0}\right) R\,\mathrm{d}\theta = \dot{q}\pi R b = \bar{q}\pi R, \qquad [8]$$

where $\bar{q}$ is the average wall heat flux.

Solutions

The present problem is solved by the integral method. The integration of [1] with the boundary condition [2] gives

$$\frac{1}{R^2}\frac{\mathrm{d}^2}{\mathrm{d}\theta^2}\left(\int_0^b T_{\mathrm{w}}\,\mathrm{d}y_{\mathrm{w}}\right) - \frac{\partial T_{\mathrm{w}}}{\partial y_{\mathrm{w}}}\bigg|_{0} + \frac{\dot{q}b}{K_{\mathrm{w}}} = 0. \qquad [9]$$

Integrating [3] twice with the boundary condition [5] we obtain the necessary velocity profile. Integrating [4] together with [6] and the velocity profile obtained, we now have

$$-K_{\mathrm{G}} \frac{\partial T_{\mathrm{G}}}{\partial y_{\mathrm{G}}}\bigg|_{0} = \frac{g(\rho_{\mathrm{L}} - \rho_{\mathrm{G}})\rho_{\mathrm{G}} h_{\mathrm{LG}}}{12\mu_{\mathrm{G}}} \frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}\theta}(\delta^3 \sin\theta). \qquad [10]$$

The temperature distribution in the tube wall, T_w, is assumed to be

$$T_w = \alpha_\theta\left[\left(\frac{y_w}{b}\right) - \frac{1}{2}\left(\frac{y_w}{b}\right)^2\right] + \beta_\theta, \qquad [11]$$

where α_θ and β_θ are functions of θ only. The radial temperature distribution given in [11] was obtained first by integrating [1] (neglecting the circumferential term), with the boundary condition [2]. To accommodate the circumferential variation of temperature within the tube wall, functions α_θ and β_θ are then introduced. T_G is obtained by integrating [4] with the boundary condition [15]:

$$T_G = \gamma_\theta\left[1 - \left(\frac{y_G}{\delta}\right)\right] + T_{sat}, \qquad [12]$$

where γ_θ is a function of θ only.

From [11], it is seen that $\alpha_\theta = (b/K_w)q_\theta$, where q_θ is defined as

$$q_\theta = K_w \left.\frac{\partial T_w}{\partial y_w}\right|_0$$

and

$$\beta_\theta = T_{w, y_w = 0}.$$

It can also be seen from [12] that

$$\gamma_\theta = \frac{\delta}{K_G} q_\theta,$$

where q_θ is defined as

$$q_\theta = -K_G \left.\frac{\partial T_G}{\partial y_G}\right|_0.$$

Applying the boundary conditions [7] and [8] to [11] and [12] yields the following relationships between α_θ, β_θ and γ_θ:

$$\beta_\theta = \frac{K_w}{K_G}\frac{\delta}{b}\alpha_\theta + T_{sat}, \qquad [13]$$

$$\gamma_\theta = \frac{K_w}{K_G}\frac{\delta}{b}\alpha_\theta \qquad [14]$$

and

$$\int_0^\pi \alpha_\theta \, d\theta = \frac{\dot{q}b^2}{K_w}\pi = \frac{\bar{q}b}{K_w}\pi. \qquad [15]$$

Therefore, the temperature distributions for T_w and T_G are:

$$T_w = \alpha_\theta\left[\left(\frac{y_w}{b}\right) - \frac{1}{2}\left(\frac{y_w}{b}\right)^2\right] + \frac{K_w}{K_G}\frac{\delta}{b}\alpha_\theta + T_{sat} \qquad [16]$$

and

$$T_G = \frac{K_w}{K_G}\frac{\delta}{b}\alpha_\theta\left[1 - \left(\frac{y_G}{\delta}\right)\right] + T_{sat}. \qquad [17]$$

Substituting [16] and [17] into [9] and [10] respectively, we obtain the following ordinary differential equations:

$$\frac{1}{R^2}\frac{d^2}{d\theta^2}\left[\left(\frac{b}{3} + \frac{K_w}{K_G}\delta\right)\alpha_\theta\right] - \frac{\alpha_\theta}{b} + \frac{\dot{q}b}{K_w} = 0 \qquad [18]$$

and

$$\frac{K_w}{b}\alpha_\theta = \frac{g(\rho_L - \rho_G)\rho_G h_{LG}}{12\mu_G}\frac{1}{R}\frac{d}{d\theta}(\delta^3 \sin\theta). \qquad [19]$$

We now introduce the following dimensionless parameters:

$$\Delta = \left(\frac{\delta}{R}\right)B^{-1/3}, \quad \text{dimensionless thickness of vapor film;} \tag{20}$$

$$\phi = \frac{\theta}{\pi}, \quad \text{dimensionless angle co-ordinate;} \tag{21}$$

$$\eta = \left[\frac{\alpha_\theta}{\left(\frac{\dot{q}b^2}{K_w}\right)}\right]\left(\frac{\delta}{R}\right)B^{-1/3} = \frac{(T_{w,y_w=0} - T_{sat})}{\left(\frac{\dot{q}bR}{K_G}\right)}B^{-1/3}, \quad \text{dimensionless outside wall superheat;} \tag{22}$$

$$H = \left(\frac{RK_G}{bK_w}\right)B^{-1/3}, \quad \text{dimensionless relative conduction parameter in film boiling;} \tag{23}$$

where

$$B = \frac{\mu_G}{g(\rho_L - \rho_G)\rho_G h_{LG}}\frac{\dot{q}b}{R^2}, \quad \text{dimensionless film boiling parameter.} \tag{24}$$

The initial conditions for [18] and [19] are as follows:

$$\Delta(0) = \Delta_0, \quad \left.\frac{d\Delta}{d\phi}\right|_0 = 0, \quad \left.\frac{d\eta}{d\phi}\right|_0 = 0, \tag{25}$$

where the value of Δ_0 is considered as finite, but not determinable. Integrating now [18] and [19] with [25], together with the assumption that $(K_w/K_G) \gg (b/3\delta)$,† will result in

$$\frac{d\eta}{d\phi} = \pi^2 H\left[\frac{\Delta^3 \sin(\pi\phi)}{12\pi} - \phi\right] \tag{26}$$

and

$$\eta = \frac{\Delta}{12\pi}\frac{d}{d\phi}[\Delta^3 \sin(\pi\phi)]. \tag{27}$$

Boundary conditions are taken from [25] and [15] together with [19]:

$$\left.\frac{d\Delta}{d\phi}\right|_0 = 0, \quad \lim_{\phi \to 1}\{\Delta^3 \sin(\pi\phi)\} = 12\pi. \tag{28}$$

The vapor film thickness, δ, the wall heat flux, q, the degree of superheating, $(T_{w,y_w=0} - T_{sat})$, the heat transfer coefficient, h, and Nusselt number, Nu, are calculated from [26]–[28] (see the appendix).

Local Values

The local values can now be expressed as follows.

(1) Vapor film thickness

$$\delta = RB^{1/3}\Delta. \tag{29}$$

(2) Wall heat flux

$$q_\phi = K_w \left.\frac{\partial T_w}{\partial y_w}\right|_0 = \dot{q}b\left(\frac{\eta}{\Delta}\right) = \bar{q}\left(\frac{\eta}{\Delta}\right). \tag{30}$$

(3) Degree of superheating

$$T_{w,y_w=0} - T_{sat} = \left(\frac{\dot{q}bR}{K_G}\right)B^{1/3}\eta = \left(\frac{\bar{q}R}{K_G}\right)B^{1/3}\eta. \tag{31}$$

†The practical value of $(b/3\delta)$ lies between $0 \sim 10$ ($\delta/R \simeq 0.01 \sim 0.1$ and $b/R \simeq 0 \sim 0.3$), and the order of (K_w/K_G) is about 10^3.

(4) Nusselt number

Two different Nusselt numbers may be defined as:

$$\text{(i) } \mathrm{Nu}_{\phi,\mathrm{a}} = \frac{h_{\phi,\mathrm{a}}(2R)}{K_\mathrm{G}} = 2B^{-1/3}\left(\frac{1}{\Delta}\right), \qquad [32]$$

where

$$h_{\phi,\mathrm{a}} = \frac{q_\phi}{(T_{w,y_w=0} - T_{\mathrm{sat}})};$$

and

$$\text{(ii) } \mathrm{Nu}_{\phi,\mathrm{b}} = \frac{h_{\phi,\mathrm{b}}(2R)}{K_\mathrm{G}} = 2B^{-1/3}\left(\frac{1}{\eta}\right), \qquad [33]$$

where

$$h_{\phi,\mathrm{b}} = \frac{\bar{q}}{(T_{w,y_w=0} - T_{\mathrm{sat}})}.$$

Average Values

Two average Nusselt numbers corresponding to the local values are defined as follows:

$$\text{(i) } \overline{\mathrm{Nu}}_\mathrm{a} = \frac{\bar{h}_\mathrm{a}(2R)}{K_\mathrm{G}} = 2B^{-1/3}\int_0^1 \frac{1}{\Delta}\,\mathrm{d}\phi; \qquad [34]$$

and

$$\text{(ii) } \overline{\mathrm{Nu}}_\mathrm{b} = \frac{\bar{h}_\mathrm{b}(2R)}{K_\mathrm{G}} = 2B^{-1/3}\int_0^1 \frac{1}{\eta}\,\mathrm{d}\phi. \qquad [35]$$

RESULTS AND DISCUSSION

The results of the present analysis are all presented in dimensionless parameters. Comparison is not made with experimental studies but with that of Bromley's (1950) analytical solution for zero vapor velocity at the vapor–liquid interface for an isothermal cylinder, which corresponds to the solution for the case of $H = 0$ in the present analysis (see the appendix).

In figure 2, the inverse correlations of the dimensionless thickness of the local vapor film are given for $H = 0$ to $H \to \infty$. The solution for the case of $H = 0$ corresponds to Bromley's analytical solution with zero vapor velocity at the vapor–liquid interface for an isothermal horizontal cylinder. It can be seen that, the larger the value of H,† the thinner the vapor film thickness. This implies that a higher value of H will result in a higher Nusselt number because the thermal resistance becomes smaller with the decreasing value of the vapor film thickness. The maximum differences are seen at the forward stagnation point for all values of H. For the cases of $H = 0$ and $H = \infty$, it is $<8\%$, indicating that the effect of H on the vapor film thickness is relatively insignificant. $1/\Delta$ becomes zero at $\phi = 1$, and this is because the characteristics of the solution for the problem dictate that the vapor film thickness at $\phi = 1$ be infinite.

The reciprocals of the dimensionless outside wall superheat which are equivalent to the local Nusselt numbers, $\mathrm{Nu}_{\phi,\mathrm{b}}$, [33], are shown in figure 3. The case of $H = 0$ corresponds to the uniform outside wall temperature solution (i.e. Bromley's solution); and $H \to \infty$ to the uniform outside wall heat flux solution. Non-uniformity of the dimensionless outside wall superheat increases with an increase in the value of H. It can be seen that the effect of H is significant.

As pointed out by Baughn (1978), in experimental studies of convective heat transfer in a circular tube, the measurements are usually made at the outside of the tube to determine the inside wall thermal boundary condition which is then inferred by solving the heat conduction problem in the

†The value of H lies between 0.4 and 2.5 for an S.S.-304 stainless tube of o.d. = 25.4 mm, $b/R = 0.05$ to 0.3, working fluid of Freon-113 at 1 atm and average heat flux of 30 kW/m^2.

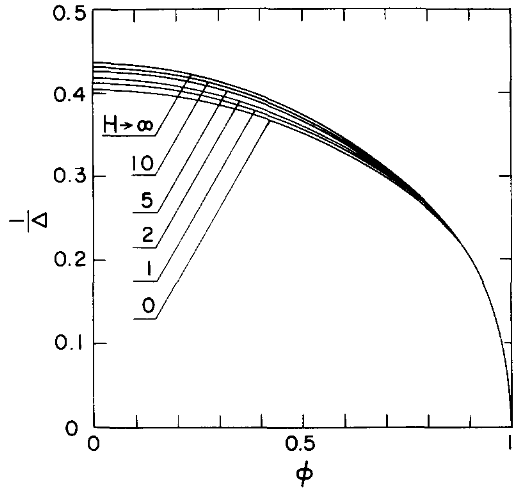

Figure 2. Vapor film thickness.

wall. Figure 3 illustrates that care must be taken to attach thermocouples in such situations so as to make meaningful temperature measurement.

Figure 4 shows the circumferential wall heat flux distribution. Non-uniformity of $q_\phi/\bar{q}$ increases with a decrease in the value of H and the extreme is seen at $H = 0$, i.e. Bromley's solution for an isothermal ($1/\eta$ = const) horizontal cylinder. Since the vapor film thickness is infinite at $\phi = 1$, the ratio $q_\phi/\bar{q}$ becomes zero. The results shown in figure 4 are subjected to the condition that the total wall heat transfer rate is the same for all cases of H, i.e. [8].

The solution by the present analysis for $1/\eta$ = const and $H = 0$ is exactly identical to the Bromley solution for an isothermal horizontal cylinder with zero vapor velocity at the vapor–liquid interface (see the appendix). The functional relationship between the dimensionless relative conduction parameter, H, the dimensionless thickness of the vapor film, Δ, and the dimensionless wall superheat, η, at the forward stagnation point is illustrated in figure 5. The values of Δ at $\phi = 0$ and η at $\phi = 0$ converge to the same constant value because, as shown in the appendix. Δ is exactly the same as η for the case of $H \to \infty$. Thus, from [34] and [35], it can be seen that $\overline{Nu}_a = \overline{Nu}_b$. For the values of H from 0 to ∞, the maximum variation of Δ was only about 7%, while that of η was about 25%.

Figure 6 shows the distribution of the two average Nusselt numbers as a function of H. At $H = 0$ (the case of a uniform outside wall temperature), there is no difference between the values of $\overline{Nu}_a$

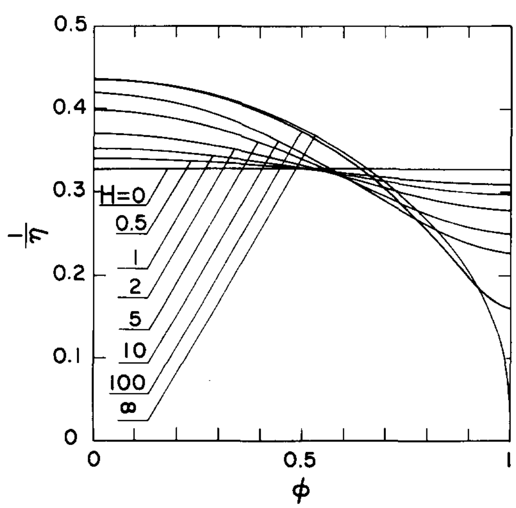

Figure 3. Wall superheat.

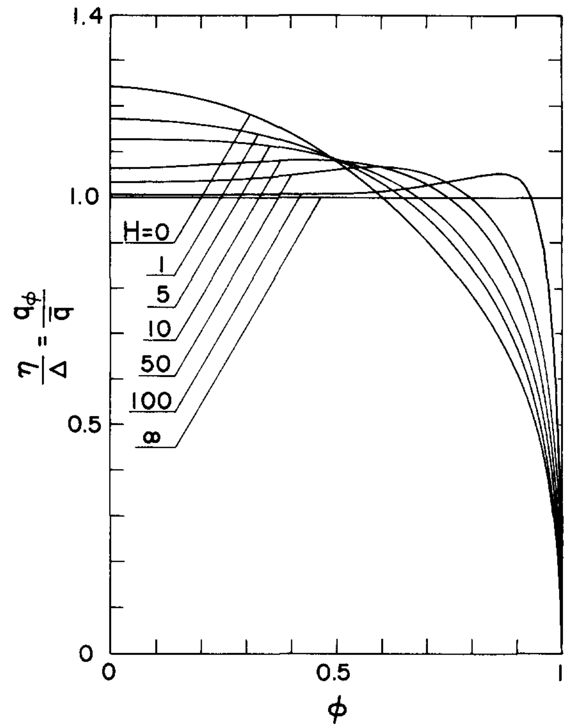

Figure 4. Local wall heat flux.

and $\overline{\mathrm{Nu}}_b$, which was expected. As the value of H increases, the difference between $\overline{\mathrm{Nu}}_a$ and $\overline{\mathrm{Nu}}_b$ increases, and they eventually converge to the same constant value. The maximum difference in the values for $H = 0$ to $H \to \infty$ is about 6%.

The effect of H on the average Nusselt numbers starts to appear as H becomes >0.1. H in terms of the Biot number, Bi can be written as

$$H = \frac{\mathrm{Bi}}{2\left(\frac{b}{R}\right) \cdot C_j},$$

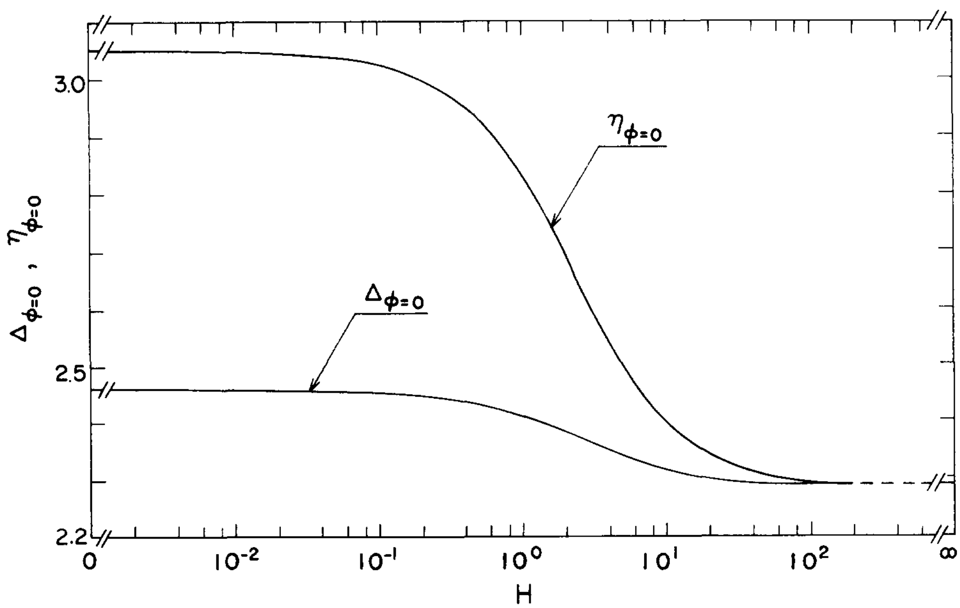

Figure 5. Effects of H on vapor film thickness and wall superheat at the forward stagnation point.

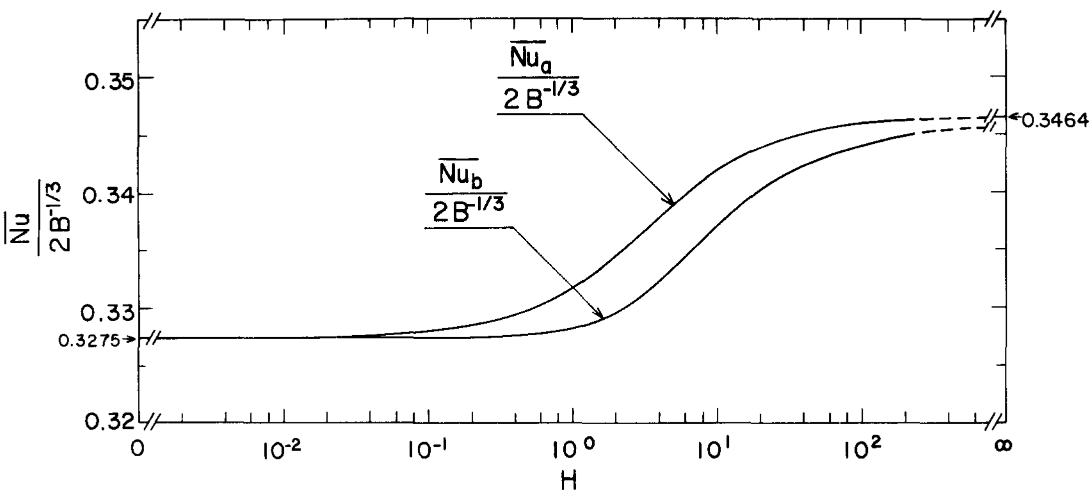

Figure 6. Effect of H on the average Nusselt numbers.

where

$$\mathrm{Bi} = \frac{\bar{h}(2R)}{K_w}$$

and

$$C_j = \int_0^1 \left(\frac{1}{j}\right) d\phi, \quad j = \Delta \text{ or } \eta.$$

The relationship between H and Bi as a function of (b/R) is shown in figure 7 as an illustration. The practical range of (b/R) is about 0.05–0.3.

From figures 6 and 7, it can be concluded that the effect of relative circumferential conduction on film pool boiling starts to appear when the values of Bi become >0.003 for $(b/R) = 0.05$ and 0.02 for $(b/R) = 0.3$.

As was discussed in conjunction with figure 3, figure 6 also illustrates that different numerical values of average h and Nu can result for the same experiment depending on the definition of average for h and Nu. It can be seen from the present analysis, for the cases of $H = 0$ and $H \to \infty$, that $\overline{\mathrm{Nu}}_a$ and $\overline{\mathrm{Nu}}_b$ have exactly the same value. For $0 \leqslant H \leqslant \infty$, however, both are not the same

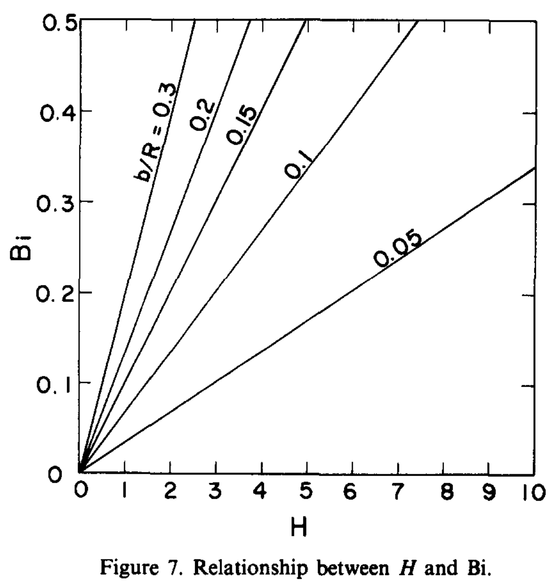

Figure 7. Relationship between H and Bi.

and $\overline{\mathrm{Nu}}_a$ is always $>\overline{\mathrm{Nu}}_b$. In experimental studies, usually only the value of $\overline{\mathrm{Nu}}_b$ can be deduced from the average wall heat flux, $\bar{q}$, and the distribution of the local wall superheat, $(T_{w,y_w=0} - T_{sat})$. Therefore, one has to be very careful that the experimentally obtained average Nusselt numbers are compared correctly with the corresponding values from the theoretical analysis.

It is important to point out that the dimensionless number H is an additional parameter to describe conjugated heat transfer problems on film pool boiling and is similar to those deduced by Lee & Kakade (1976) from the governing differential equations and by Chida & Katto (1976) from a vectorial dimensional analysis. This implies that additional dimensionless parameters are always needed to describe conjugated heat transfer problems, be they steady or unsteady.

CONCLUSIONS

The study leads to the following conclusions:

1. For $0 \leqslant H \leqslant \infty$, the effect of H on the vapor film thickness is relatively small, while the dimensionless outside wall superheat, η, which equivalent to the local Nusselt number, $\mathrm{Nu}_{\phi,b}$, is significantly affected by it.
2. For $0 \leqslant H \leqslant \infty$, the maximum variation of the vapor film thickness at the forward stagnation point was only about 7%, while that of the outside wall superheat was about 25%.
3. For $H \leqslant 0.01$, there is no difference between the values of $\overline{\mathrm{Nu}}_a$ and $\overline{\mathrm{Nu}}_b$. As the value of H increases, the difference between $\overline{\mathrm{Nu}}_a$ and $\overline{\mathrm{Nu}}_b$ increases and the values of $\overline{\mathrm{Nu}}_a$ and $\overline{\mathrm{Nu}}_b$ eventually converge to the same constant value.
4. Since different numerical values of average h and Nu can result for the same experiment, depending on the definition of average for h and Nu, caution must be taken in measuring the local wall surface temperature distribution, except for the case where $H = 0$.

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APPENDIX

Solutions of [26]–[28]

Transforming the variable Δ as

$$w = \left[\frac{\Delta^3 \sin(\pi\phi)}{12\pi}\right]^{4/3}, \tag{A.1}$$

[26] and [27] become

$$\frac{d\eta}{d\phi} = \pi^2 H(w^{3/4} - \phi) \tag{A.2}$$

and

$$\frac{dw}{d\phi} = \frac{4}{3}\left[\frac{\sin(\pi\phi)}{12\pi}\right]^{1/3}\eta. \tag{A.3}$$

The boundary conditions are

$$w(0) = 0, \quad w(1) = 1. \tag{A.4}$$

For the special cases of (a) $H = 0$ and (b) $H \to \infty$, we can obtain the exact solutions for [A.2]–[A.4], as given below.

(a) $H = 0$

For this case, we obtain the following uniform outside wall temperature solution:

$$\eta = \frac{3}{4}\frac{1}{\displaystyle\int_0^1 \left[\frac{\sin(\pi\phi)}{12\pi}\right]^{1/3} d\phi} \approx 3.054, \tag{A.5}$$

$$\Delta = \frac{(12\pi)^{1/3}}{\left[\displaystyle\int_0^1 \sin(\pi\phi)^{1/3}\, d\phi\right]^{1/4}} \left\{\frac{\displaystyle\int_0^\phi [\sin(\pi\phi)]^{1/3}\, d\phi}{[\sin(\pi\phi)]^{4/3}}\right\}^{1/4} \approx 3.520 \left\{\frac{\displaystyle\int_1^\phi [\sin(\pi\phi)]^{1/3}\, d\phi}{[\sin(\pi\phi)]^{4/3}}\right\}^{1/4} \tag{A.6}$$

and

$$\frac{q_\phi}{\bar{q}} = \frac{\eta}{\Delta} \approx 0.8676 \left\{\frac{[\sin(\pi\phi)]^{4/3}}{\displaystyle\int_0^\phi [\sin(\pi\phi)]^{1/3}\, d\phi}\right\}^{1/4}. \tag{A.7}$$

In this case, the two average heat transfer coefficients $\bar{h}_a$, and $\bar{h}_b$, defined by [34] and [35], have exactly the same value:

$$\bar{h}_{H=0} = \bar{h}_a = \bar{h}_b = 0.3275\left(\frac{K_G}{R}\right)B^{-1/3} = 0.4126\left[\frac{K_G^3 g(\rho_L - \rho_G)\rho_G h_{LG}}{D\mu_G \bar{q}}\right]^{1/3}, \tag{A.8}$$

where D is the outside diameter of the tube.

Since

$$\bar{q} = \bar{h}_{H=0}(T_{w,y_w=0} - T_{sat}) = \bar{h}_{H=0}\Delta T_{sat},$$

[A.8] can be rewritten as

$$\bar{h}_{H=0} = 0.5148\left[\frac{K_G^3 g(\rho_L - \rho_G)\rho_G h_{LG}}{D \cdot \mu_G \cdot \Delta T_{sat}}\right]^{1/4}. \qquad [A.9]$$

Equation [A.9] coincides with Bromley's (1950) analytical solution for zero vapor velocity at the vapor–liquid interface.

(*b*) $H \to \infty$

For this case, we obtain the following uniform outside wall heat flux solution:

$$\eta = \left[\frac{12\pi\phi}{\sin(\pi\phi)}\right]^{1/3}, \qquad [A.10]$$

$$\Delta = \eta = \left[\frac{12\pi\phi}{\sin(\pi\phi)}\right]^{1/3} \qquad [A.11]$$

and

$$\frac{q_\phi}{\bar{q}} = \frac{\eta}{\Delta} = 1. \qquad [A.12]$$

The average heat transfer coefficients are

$$\bar{h}_a = \bar{h}_b = 0.3464\left(\frac{K_G}{R}\right)B^{-1/3} = 0.4364\left[\frac{K_G^3 g(\rho_L - \rho_G)\rho_G h_{LG}}{D\mu_G\bar{q}}\right]^{1/3}. \qquad [A.13]$$

(*c*) $0 \leqslant H \leqslant \infty$

For the case, $0 \leqslant H \leqslant \infty$, the system given by [A.2]–[A.4] becomes a two-point boundary value problem with the initial value of η unknown at $\phi = 0$. This was solved numerically using the "shooting method" (Conte & De Boor 1972) with the numerical integration by the fourth-order Runge–Kutta method. However, the Runge–Kutta method is not applicable for the present system of [A.2]–[A.4] near the forward stagnation point ($\phi = 0$), for which we used a series solution in the region of $\phi = 0$ to $\phi = 0.01$. The numerical calculation procedures by the shooting method are:

(1) choose an initial value of η;
(2) calculate the values of η and w from the series solution, with the assumed value of η and $w = 0$ at $\phi = 0$ in the region of $\phi = 0$ to $\phi = 0.01$;
(3) integrate the differential equations [A.2] and [A.3] from $\phi = 0.01$ to $\phi = 1$ using the Runge–Kutta method with a step size of 0.001;
(4) the procedure is repeated until the calculated value of w at $\phi = 1$ satisfies the following condition

$$|w_{\phi=1} - 1| \leqslant 5 \cdot 10^{-6}.$$

Numerical solutions of the system [A.2]–[A.4] were obtained in the range of $H = 0.001$ to $H = 200$.